$$\Phi_j^n = \tfrac{1}{2}\left[\left(\frac{x_{j+1}^n - x_j^n}{v_{j+\frac{1}{2}}^n}\right) + \left(\frac{x_j^n - x_{j-1}^n}{v_{j-\frac{1}{2}}^n}\right)\right] \tag{5.3}$$

$= \text{average mass at } j.$

At the left boundary

$$\Phi_o^n = (\tfrac{1}{2})(x_1^n - x_o^n)/v_{\frac{1}{2}}^n \tag{5.4}$$

The new coordinate is given by:

$$x_j^{n+1} = x_j^n + u_j^{n+\frac{1}{2}}\,\Delta t. \tag{5.5}$$

2. Continuity equation:

$$v_{j+\frac{1}{2}}^{n+1} = v_{j+\frac{1}{2}}^n + \Delta t\left(\frac{\rho o}{m}\right)_{j+\frac{1}{2}}\left(u_{j+1}^{n+\frac{1}{2}} - u_j^{n+\frac{1}{2}}\right) \tag{5.6}$$

where

$$m_{j+\frac{1}{2}} = \rho_{j+\frac{1}{2}}^o\left(x_{j+1}^o - x_j^o\right) = \text{mass in the cell } j+\tfrac{1}{2}, \tag{5.7}$$

$\rho o = $ initial density.

3. Linear viscosity:

$$q_{j+\frac{1}{2}}^{n+\frac{1}{2}} = C_L\,\rho_{j+\frac{1}{2}}^o\,\eta_{j+\frac{1}{2}}^{n+\frac{1}{2}}\left|u_{j+1}^{n+\frac{1}{2}} - u_j^{n+\frac{1}{2}}\right| \text{for}\begin{cases} u_{j+1}^{n+\frac{1}{2}} < u_j^{n+\frac{1}{2}} \\ v_{j+\frac{1}{2}}^{n+1} < v_{j+\frac{1}{2}}^n \end{cases} \tag{5.8}$$

$= 0$ otherwise.

Here

$$\eta_{j+\frac{1}{2}}^{n+\frac{1}{2}} = 2v_o/\left(v_{j+\frac{1}{2}}^{n+1} + v_{j+\frac{1}{2}}^n\right). \tag{5.9}$$

4. Constitutive relations:

The relaxation equation:

$$\alpha_{j+\frac{1}{2}}^{n+1} = \alpha_{j+\frac{1}{2}}^{n} + \left(\frac{\alpha^{eq}-\alpha}{\tau}\right)_{j+\frac{1}{2}}^{n} \Delta t. \qquad (5.10)$$

τ is the characteristic relaxation time and is assumed to be constant.

The specific volume of the first phase is:

$$v_{1,j+\frac{1}{2}}^{n+1} = v_{j+\frac{1}{2}}^{n+1} - (v_2 - v_1)\alpha_{j+\frac{1}{2}}^{n+1} \qquad (5.11)$$

Temperature calculation:

$$T_{j+\frac{1}{2}}^{n+1} = T_{j+\frac{1}{2}}^{n} + [CT]_{j+\frac{1}{2}}^{n}(v_{j+\frac{1}{2}}^{n+1} - v_{j+\frac{1}{2}}^{n})$$

$$- \left(\frac{q}{C_v}\right)_{j+\frac{1}{2}}^{n+\frac{1}{2}}(v_{j+\frac{1}{2}}^{n+1} - v_{j+\frac{1}{2}}^{n}) \qquad (5.12)$$

where $C = -(1/C_v)(\partial p/\partial T)_v$ (convenient for a mixed phase)

or $C = -\Gamma/v$ (convenient for a single phase), and Γ is the Grüneisen coefficient. The value of C_v depends on the phase region as described in the last section. The formula for $C_{v,m}$ in a mixed phase is given in Appendix II.

Equation of state:

$$p_{j+\frac{1}{2}}^{n+1} = [p(v_1,T)]_{j+\frac{1}{2}}^{n+1} \qquad (5.13)$$

where $p(v_1,T)$ is given by Eq. (4.5).

The boundary between a single and a two-phase region is distinguished by the transition pressure p_M, at which the relative volume of the first phase is given by v_A.